Computer-Aided Microwave Circuit Analysis by a Computerized Numerical Smith Chart

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Abstract—**The Smith chart, while being a very useful graphical tool for the manual analysis and design of microwave circuits, is not suitable for computer manipulation. A computerized Smith chart represented by a numerical matrix is developed. In addition to the fact that this numerical matrix is very convenient for computer storage and efficient for algorithmic operations, it retains the locality relationships between impedances and reflection coefficients on a graphical Smith chart.**

I. Introduction

THE analysis and design of microwave circuits are generally tedious in analytical form. The Smith chart provides a very useful graphical tool for these problems. Typical problems that can be analyzed using a Smith chart include impedance matching, stability analysis, etc. [1]. Analysis of and design results for these problems are conveniently obtained by tracing constant-resistance circles, constant-conductance circles, and their intersections on the Smith chart. Though the use of a Smith chart is intuitively convenient, its manual interpretation can be error prone. Also, its graphical representation of the reflection coefficient plane (Γ-plane), while suitable for manual operations, is not adequate for computer manipulation in a computer-aided design (CAD) environment.

Recently, CAD methodology has established its indispensable role in microwave circuit engineering activities. In a CAD system, the mapping of impedances from the z-plane to the Γ-plane has to be represented by their transformation equations that can be manipulated by a computer. Though these equations theoretically contain the same information as a Smith chart, they no longer contain the mental picture which naturally resides in a graphical representation. The equations are thus less intuitively comprehensible to a designer who has to utilize the analysis results. Furthermore, compared to the tracing of lines and intersections on a Smith chart, relatively complicated algorithms are needed to use the transformation equations.

This letter provides a scheme to computerize the Smith chart in a CAD system. Two criteria are set for this computerized Smith chart generation scheme. First, the transformation from the graphical Smith chart to a computer database must be simple and the result must be efficient for computer manipulation. Second, the computerized Smith chart must facilitate the development of algorithms to perform analysis and design tasks.

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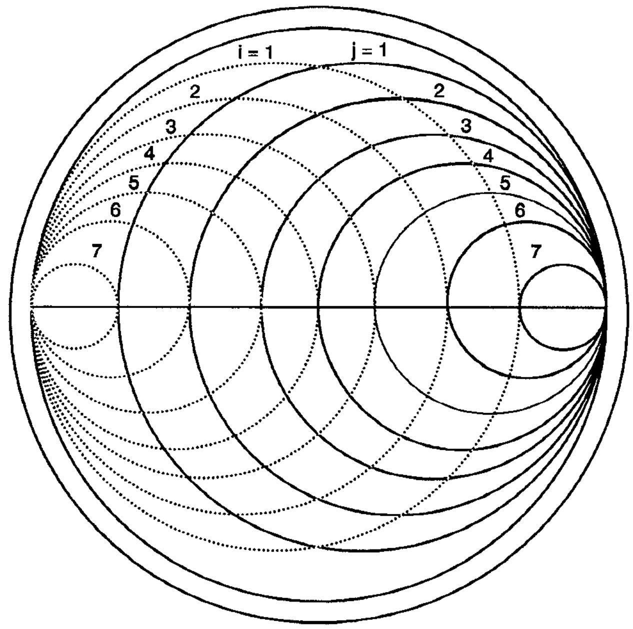

Fig. 1. Example Y–Z Smith chart serving as the input to the mapping scheme.

Section II of this letter presents a scheme to map a normalized impedance and admittance Smith chart (Y–Z Smith chart) into a simple numerical matrix. Example algorithms developed to design impedance matching circuits and stability improving circuits are shown in Section III.

II. Mapping of a Y–Z Smith Chart

A Y–Z Smith chart is shown in Fig. 1. For clarity, only constant-resistance and constant-conductance circles are provided in Fig. 1. Resistance and conductance circles are shown with solid and dotted lines, respectively. It should be noted that the pure resistance/conductance line, which is a special case of the constant-resistance/conductance circles, is also included in this Y–Z Smith chart. Though only a sufficient number of circles for the explanation of this novel representation are provided in Fig. 1, the scheme to be described can be readily extended to any desired precision by adding circles to the Y–Z Smith chart. For example, a Y–Z Smith chart with 50 resistance circles and 50 conductance circles will provide the precision required by many practical applications.

Since most operations performed with a Smith chart are carried out by tracing circles and reading the impedance/admittance values at their intersections, the database of its computerized counterpart must be designed to facilitate these activities. As seen in Fig. 1, the resistance and conductance circles intersect with each other and form a set of cross-over points on the Y–Z Smith chart. The impedance/admittance values at these cross-over points can be found by identifying their locations on the Y–Z Smith chart. For the desired precision, it is reasonable to assume that there are identical numbers of resistance and conductance circles. The modification of this mapping scheme is straightforward if this assumption has to be removed for any reasons.

In this new scheme, the Y–Z Smith chart with n resistance circles and n conductance circles is mapped into an n-by-n numerical matrix S. As shown in Fig. 1 ($n = 7$), if the conductance and resistance circles are labeled, beginning with the largest circles, by i and j, $1 \leq i, j \leq n$, respectively, a cross-over point then acquires a pair of coordinates $[i, j]$. The impedance/admittance represented by a cross-over point is denoted by $Z[i, j]$. It can be observed from Fig. 1 that, the points on the pure resistance/conductance line are created by two circles tangent to each other. However, a pair of resistance and conductance circles cross over at two points, and thus, there are two impedance/admittance values associated with each pair of coordinates $[i, j]$. In the following discussion, the impedances on the upper and lower half planes of the chart are distinguished by identifying them as $Z[i, j]$ and $Z'[i, j]$, respectively. An impedance $Z[i, j]$ (or $Z'[i, j]$) on the Y–Z Smith chart is mapped to an element $S[u, v]$ of matrix S, where u and v are the row and column numbers of S, respectively. This mapping is performed by the transformation described in the following pseudo-code listing.

```
FOR i = 1 TO n
  FOR j = 1 TO (n + 1 − i)
    IF (i + j) = (n + 1) THEN
      u = i, and v = j;
      S[u, v] = Z[i, j];
    ELSE
      u1 = i, v1 = j, u2 = (n + 1) − i, v2 = i + j;
      S[u1, v1] = Z[i, j]; S[u2, v2] = Z'[i, j];
    END IF
  END j
END i.
```

The significance of this mapping will be explained with respect to the specific example Y–Z Smith chart shown in Fig. 1. The result matrix S created by this mapping is provided in (1), which shows the mapped Smith chart impedances $Z_{ij} = Z[i, j]$ or $Z'_{ij} = Z'[i, j]$. The conductance and resistance circles in the Y–Z Smith chart are mapped into rows and columns of matrix S, respectively. The points on the pure resistance/conductance line of the Y–Z Smith chart are mapped into the reverse diagonal of matrix S (bold faced) and $Z_{44} = Z[4, 4]$ corresponds to $Z_o = 1$ in the Y–Z Smith chart. The data structure of S can now be easily represented in a CAD environment:

$$S = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{14} & Z_{15} & Z_{16} & \mathbf{Z_{17}} \\ Z_{21} & Z_{22} & Z_{23} & Z_{24} & Z_{25} & \mathbf{Z_{26}} & Z'_{61} \\ Z_{31} & Z_{32} & Z_{33} & Z_{34} & \mathbf{Z_{35}} & Z'_{51} & Z'_{52} \\ Z_{41} & Z_{42} & Z_{43} & \mathbf{Z_{44}} & Z'_{41} & Z'_{42} & Z'_{43} \\ Z_{51} & Z_{52} & \mathbf{Z_{53}} & Z'_{31} & Z'_{32} & Z'_{33} & Z'_{34} \\ Z_{61} & \mathbf{Z_{62}} & Z'_{21} & Z'_{22} & Z'_{23} & Z'_{24} & Z'_{25} \\ \mathbf{Z_{71}} & Z'_{11} & Z'_{12} & Z'_{13} & Z'_{14} & Z'_{15} & Z'_{16} \end{bmatrix}. \quad (1)$$

III. Application Examples

The computerized Smith chart represented in a matrix format can be easily manipulated to perform desired operations. Two typical examples of using this data structure are provided here. The discussion will concentrate on the identification of locations on the computerized Smith chart. A detailed description showing the simple steps in using these results can be found in Gonzalez [1] and will not be elaborated here.

Example 1: Given an impedance $Z[i, j]$, the following equation can be used to determine the impedance matching networks required to match it to $Z_o = 1$,

$$[u_1, v_1] = [i_o, j]; \qquad [u_2, v_2] = [n + 1 - i_o, i_o + j], \quad (2)$$

where $[i, j]$ and $[i_o, j_o]$ are the locations of Z and Z_o on the graphical Y–Z Smith chart, respectively, n is the number of resistance/conductance circles, and $[u_1, v_1]$ and $[u_2, v_2]$ indicate elements in matrix S. The difference between $Z[i, j]$ and $S[u_1, v_1]$ (or $S[u_2, v_2]$) and the difference between $S[u_1, v_1]$ (or $S[u_2, v_2]$) and Z_o will determine the topology and value of an impedance network that should be added to the circuit.

A numerical example is given here to demonstrate the use of (2). In Fig. 2, suppose an impedance located at $[i, j] = [6, 1]$ is to be matched with $Z_o = Z_{44}$, applying (2) will yield two locations $[u_1, v_1] = [4, 1]$ and $[u_2, v_2] = [4, 5]$ in matrix S. It can be readily verified that $S[4, 1] = Z_{41}$ and $S[4, 5] = Z'_{41}$ contain the two impedances located at $[i, j] = [4, 1]$ on the graphical Y–Z Smith chart. The impedance matching network can now be determined by the difference between Z_{61} and Z_{41} (or Z'_{41}) and the difference between Z_{41} (or Z'_{41}) and Z_{44}.

Example 2: Given a certain circuit with $K < 1$ (i.e., potentially unstable), the calculation of a stability circle can be used to identify a resistance circle or a conductance circle as the boundary of the unconditionally stable region. If a resistance circle is selected, the following equation is provided for determining the resistance that has to be added to stabilize the circuit:

$$[u_1, v_1] = [n + 1 - j, j], \quad (3)$$

where j is the resistance circle selected, n is the number of resistance/conductance circles in the Y–Z Smith chart, and $[u_1, v_1]$ is a location on the reverse diagonal of matrix S. Since the reverse diagonal of matrix S represents the pure resistance line in the Y–Z Smith chart, the value of $S[u_1, v_1]$ indicates the resistance that is needed to be added for the purpose of stabilization.

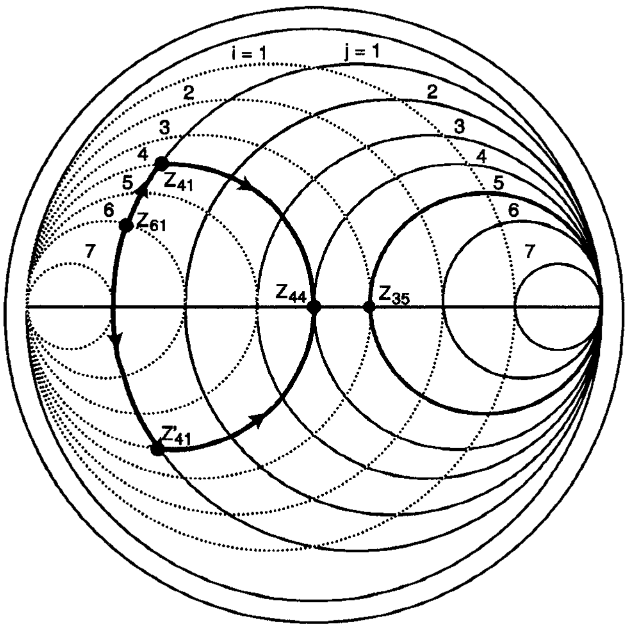

Fig. 2. A Y–Z Smith chart demonstrating the results in Examples 1 and 2.

Similarly, if instead, a conductance circle is identified as the boundary of the stable region, the following equation can be used to stabilize the circuit:

$$[u_2, v_2] = [i, n + 1 - i], \quad (4)$$

where i is the conductance circle selected, n is the number of resistance/conductance circles in the Y–Z Smith chart, and $[u_2, v_2]$ is a location on the reverse diagonal of matrix S. The value of $S[u_2, v_2]$ indicates the conductance that is required to be added to stabilize the circuit.

A numerical example is given here to demonstrate the use of these equations. If a resistance circle $j = 5$ is picked, (3) will provide $[u_1, v_1] = [3, 5]$. $S[3, 5] = Z_{35}$ then contains the resistance that is needed for stabilization. This result is also shown in Fig. 2.

IV. Summary

In summary, a mapping scheme is provided to transform a graphical Y–Z Smith chart into a matrix that can be efficiently stored and conveniently manipulated by a computer system. The advantage of this computerized Smith chart is that while simple computer algorithms can be easily developed to perform operations such as impedance matching and stabilization, the intuition naturally provided by a graphical representation is reserved for human users.

References

[1] G. Gonzalez, *Microwave Transistor Amplifiers Analysis and Design*. Englewood Cliffs, NJ: Prentice-Hall, 1984, pp. 42–80, 95–102.